Detailed Notes on Learning Curves

🌞 Overview — What Is a Learning Curve?

The learning curve (or experience curve) represents how labor efficiency improves as workers gain experience through repetition of tasks. It shows that as workers gain experience producing a product or performing a task, efficiency increases and time per unit decreases. In simple terms: "The more you produce, the faster and cheaper you become." As the cumulative number of units produced doubles, the average time per unit decreases by a constant percentage (called the learning rate). A learning curve is a curve showing the reduction in labor time per unit as cumulative production increases.

Learning curves apply to labor-intensive processes where repetition improves efficiency. They are particularly relevant in cost management for estimating labor costs and forecasting.

⚙️ 1. Key Concepts

- **Efficiency and Experience**: Each repetition of a task improves efficiency. Labor time per unit decreases as cumulative production increases. Productivity improves rapidly at first, then slows and stabilizes over time.
- **Terminology**:

Term	Meaning
Learning Curve	A curve showing the reduction in labor time per unit as cumulative production increases.
Learning Rate (LC)	The percentage of the previous average time that the new cumulative average time will be after production doubles.
Doubling (n)	Each time cumulative production doubles (e.g., from 1 to 2, 2 to 4, 4 to 8, etc.).

💼 2. Use and Importance

Learning curve analysis helps management in:

- 📊 Planning, Budgeting, and Forecasting
- 💰 Estimating labor costs for production and contract bids
- 📈 Cost control and performance evaluation
- 🔧 Setting standards for new production processes

It estimates:

- Future labor hours
- Long-term cost behavior
- Expected cost reductions as output doubles

Learning curves are useful for cost estimation, budgeting, pricing, and performance evaluation. When overhead is applied based on labor hours, costs may need segregation into fixed costs and variable costs (using methods like High-Low or Regression Analysis). In manufacturing or service contexts, it supports bids/contracts by predicting efficiency gains.

🧩 3. Two Learning Curve Models

Model	Description	Estimates	CMA Exam Focus
1️⃣ Cumulative Average-Time Model	Assumes the average time per unit declines by a constant rate each time cumulative output doubles.	Average time per unit for all units produced	✅ Tested
2️⃣ Incremental Unit-Time Model	Assumes the time for the most recent unit declines by a constant rate as output doubles.	Time for the last unit produced	❌ Not tested

The Cumulative Average-Time Model is the only one tested on the CMA exam.

Scenario: A factory produces metal widgets. The first unit takes **10 hours** to make. The company expects a **learning rate (LC) of 80%**, meaning each time cumulative production doubles, the average time per unit drops to 80% of the previous average.

🔢 Step-by-Step Breakdown

Doubling	Units Produced	Formula Used	Cumulative Avg. Time per Unit	Total Time
0	1	10×(0.80) ^0	10.00 hrs	10.00 hrs
1	2	10×(0.80) ^1	8.00 hrs	16.00 hrs
2	4	10×(0.80) ^2	6.40 hrs	25.60 hrs
3	8	10×(0.80) ^3	5.12 hrs	40.96 hrs

🧠 Interpretation

- As production **doubles**, the **average time per unit drops** due to increased worker efficiency.
- Even though each unit takes less time, the **total time still increases** because more units are being made.
- This helps managers **forecast labor costs**, **set realistic production schedules**, and **price contracts competitively**.

📌 CMA Tip

If asked to find the time for producing units 5–8:

- Total time for 1–8 units = 40.96 hrs

- Total time for 1–4 units = 25.60 hrs
- Time for units 5–8 = 40.96 − 25.60 = **15.36 hrs**
- Average per unit = 15.36 ÷ 4 = **3.84 hrs**

🔣 4. Mathematical Expression

For the Cumulative Average-Time Model:

Estimated Cumulative Average Time per Unit = Time for 1st Unit × (LC)^n **Estimated Total Time** = Cumulative Average Time per Unit × Total Units Produced

Where:

- LC = Learning Curve Rate (e.g., 80% = 0.80)
- n = Number of doublings (1→2, 2→4, 4→8, etc.)

Core Formulas:

- Estimated cumulative average time per unit: Average Time per Unit = $T_1 \times (LC)^n$ Where: T_1 = Time required for the first unit; LC = Learning curve percentage (in decimal form, e.g., 80% = 0.8); n = Number of doublings of cumulative production.
- Estimated total time for all units: Total Time = $T_1 \times (2 \times LC)^n$ Then, Average Time per Unit = Total Time / Total Units

Formula Relationships:

- Total Time = Average Time per Unit × Total Units
- Average Time per Unit = Total Time / Total Units
- n = $\log_2$(Total Units Produced)

🏭 Widget Manufacturing Example

Scenario:

- First unit takes 10 hours
- LC = 80%
- Total units = 8
- Doublings:

n=log2(8)=3

🔹 **Step-by-Step Calculation**

Step 1: Total Time

Total Time=10×(2×0.80)3=10×(1.6)3=10×4.096=40.96 hours

Step 2: Average Time per Unit

Average Time=40.96/8=5.12 hours

✅ **Method Comparison**

Method Used	Total Time	Avg. Time per Unit
T1×(LC)n×8	40.96 hrs	5.12 hrs
T1×(2×LC)n	40.96 hrs	5.12 hrs

Both methods yield **identical results**. Choose based on what the question asks: total time or average time.

🧠 **Understanding Doublings (n)**

What Is a Doubling?

Each time cumulative production doubles, efficiency improves and average time drops by LC%.

Total Units	Doublings (n)	Reason
1	0	No doubling
2	1	21=22^1 = 2
4	2	22=42^2 = 4
8	3	23=82^3 = 8
16	4	24=162^4 = 16

For Non-Power-of-2 Units

Use:

n=log2(Total Units)

Example: Total Units = 10

n=log2(10)≈3.32

You can **plug this decimal directly into the formula**—no rounding needed.

📐 **Applying to Non-Power-of-2 Units**

Given:

- T1=10T_1 = 10 hours
- LC=0.80LC = 0.80
- n=log2(10)≈3.32
- Total Time=10×(2×0.80)3.32=10×(1.6)3.32

Use a calculator to compute the final value.

Use **log base 2**, not log base 10.

Don't round n —use the exact decimal.

Understanding "Cumulative Average":

- Cumulative: All units produced so far
- Average: Total time ÷ Total units Cumulative Average Time per Unit = Total Time for All Units / Total Units Each time total production doubles, the cumulative average time per unit declines by the LC%.

📉 **5. Understanding the Learning Curve Percentage (LC%)**

- The LC% indicates how much time per unit is needed after each doubling of production.
- Example: 80% Learning Curve → When output doubles, average time per unit = 80% of previous average.

Example Table (80% LC, First Unit = 10 hrs):

Doubling	Units	Cumulative Avg. Time (hrs)
1st	2	10 × 0.80 = 8.00
2nd	4	10 × 0.64 = 6.40
3rd	8	10 × 0.512 = 5.12

📊 **6. Interpretation of LC Values**

LC %	Meaning	Interpretation
100%	No learning	Time per unit remains constant
< 100%	Learning occurs	Efficiency improves
70%–90%	Normal realistic range	Common in most industries
≤ 50%	Impossible	Would imply producing double output in same or less time

✅ **Valid Range**: 50% < Learning Curve Rate < 100% **Realistic LC Range**: 70%–90% (or 70%–95%, depending on process complexity).

❗ **7. Why LC ≤ 50% Is Impossible**

- At 50%, producing double output would take the same total time as the first batch — impossible. → 10 × (2 × 0.50)^1 = 10 × 1 = 10 hrs
- At 49%, total time < time for first unit — also impossible. → 10 × (2 × 0.49)^1 = 9.8 hrs
- Even 50.01% gives an unrealistic second-unit time (≈ 0.002 hrs).

Important Rule: The learning rate must always be greater than 50% and less than 100%.

- At 50%, total time for 2 units = time for first unit → impossible

❗ **When LC = 50%, Here's What Happens**

Formula:

Total Time=T1×(2×LC)n

Given:

- T1=10 hours (time for first unit)
- LC=0.50
- n=$\log_2(2)$ = 1 (doubling from 1 to 2 units) HERE n means not total number of unit, instead no.of doubleing.

Calculation:

Total Time=10×(2×0.50)1=10×1=10 hours

🧠 **Interpretation**

- You've produced **2 units**, but the **total time is still 10 hours**.
- That implies the **second unit took 0 hours**, which is **not physically possible**.
- If LC were **less than 50%**, the second unit would take **negative time**, which is even more absurd.
- Below 50%, total time for 2 units < time for first unit → impossible
- Hence, realistic learning curves are between 70%–95%, depending on the process complexity.

Limits of the Learning Rate:

LC%	Outcome	Feasibility
≤ 50%	Impossible	Violates logic (same/less total time)
50.01%–99.99%	Possible	Practical range
100%	No learning	Constant time

🧮 **8. Calculation Methods**

Method 1 — Estimated Total Time Approach (Use when question asks for total time. Saves time — step 2 may not be required.) Estimated Total Time = Time for 1st Unit × (2 × LC)^n Then: Cumulative Average Time per Unit = Estimated Total Time / Total Units

Method 2 — Estimated Cumulative Average Time Approach (Use when question asks for average time per unit.) Cumulative Average Time per Unit = Time for 1st Unit × (LC)^n Then: Total Time = Cumulative Average Time per Unit × Total Units

🧩 **Both methods yield identical results.**

Example — 80% Learning Curve (First Unit = 10 hrs):

Units	Doublings (n)	Formula	Total Time (hrs)	Avg. Time/Unit (hrs)	
1	0	$10 \times (2\times0.8)^0$	10.00	10.00	
2	1	$10 \times (2\times0.8)^1$	16.00	8.00	
4	2	$10 \times (2\times0.8)^2$	25.60	6.40	
8	3	$10 \times (2\times0.8)^3$	40.96	5.12	5.12*8 = 40.96

Interpretation: As production doubles → average time per unit decreases → total time still increases.

📈 9. Calculating Time for a Block of Units

To find time for a range of units: Time for Units X–Y = Total Time (1–Y) − Total Time (1–(X–1)) Then: Average per Unit = Time for Block / Number of Units

Example (from 80% LC table):

- 1–4 units = 25.6 hrs
- 1–8 units = 40.96 hrs
- → 5–8 units = 40.96 − 25.6 = 15.36 hrs
- → Avg./unit = 15.36 ÷ 4 = 3.84 hrs

🧠 **Note**: Cannot calculate time for individual units (like unit 7) — only blocks.

⚠️ 10. Special Case — No Learning

If LC = 100% → No learning effect: 10 × (2 × 1.00)^1 = 20 hrs (10 hrs each for 2 cars)

If LC = 80% → Learning occurs:

- 1st car = 10 hrs
- 2nd car = 6 hrs
- Total = 16 hrs
- Avg. = 8 hrs per car

Learning curves apply to labor-intensive processes where repetition improves efficiency.

🚫 11. Limits of the Learning Rate

(Repeated for emphasis from Section 6 and 7; see table above.)

📈 12. Practical Applications

Learning curve analysis is used to:

- Estimate future labor time
- Support budgeting and forecasting
- Assist in cost estimation for bids/contracts
- Evaluate efficiency trends and performance

In contexts like software development or manufacturing, it predicts how experience reduces time with each doubling.

📚 14. Summary Table

Concept	Formula / Rule	Example Output
Average time per unit	$T_1 \times (LC)^n$	9.6 min
Total time for all units	$T_1 \times (2 \times LC)^n$	2,592 hrs
Realistic LC Range	50% < LC < 100%	Common: 70%–95%
Uses	Estimating labor cost, pricing bids, budgeting, cost control	—

🧭 15. Key CMA Exam Takeaways

- ✅ Learning curve = decline in average time per unit as production doubles
- ✅ Cumulative Average-Time Model = only one tested on CMA exam
- ✅ Two calculation methods (Total-first or Average-first) → identical results
- ✅ Valid LC range: 50% < LC < 100%
- ✅ Realistic LC range: 70%–90%
- ✅ Used for: budgeting, labor estimation, forecasting, and standard-setting
- Learning curves apply to labor-intensive processes where repetition improves efficiency.
- Useful for cost estimation, budgeting, pricing, and performance evaluation.
- Always ensure the correct number of doublings when applying the formula.
- For total time, use the formula with “(2 × LC)^n”.
- For average time, use the formula with “(LC)^n”.

Additional Notes (Enhancements): In practice, learning curves can extend beyond labor to include material costs or overall experience effects (e.g., Wright's Law in aviation). For CMA purposes, focus on the cumulative average model and verify LC% feasibility to avoid common pitfalls like invalid rates. Practice with $\log_2$ calculations for non-power-of-2 units, as $n = \log_2(\text{Total Units})$ may require approximation in exams.